ABSTRACT OF PAPERS PRESENTED AT THE 14TH ANNUAL GENERAL MEETING HELD AT NEW DELHI IN JANUARY 1961

PART I

1. *Minimax Estimation of the Mean of a Normal Distribution from Contaminated Data.* *M. N. Ghosh, Institute of Agricultural Research Statistics, New Delhi* 12.

In this paper the problem of estimation of the mean of a normal distribution when the sample is contaminated, *i.e.*, there is an observation with a mean $\theta + \delta$, when the true mean to be estimated is θ. This problem has been treated by Pearson, Dixon, etc., by using criteria of rejection of outliers. Dixon also suggested the use of the mean-square error as a criterion of comparing different methods of estimation, which is equivalent to a quadratic loss function. It is shown here that a unique minimax estimate exists in this case which is also admissible. It is also shown that many other estimation problems with quadratic loss functions lead to unique minimax solutions.

2. *The Power and Relative Efficiency of Certain Tests of Homogeneity against Ordered Alternatives.* *V. J. Chacko, Forest Research Institute, Dehra Dun.*

A lower bound to the power of the Likelihood Ratio test for the equality of the means of three normal populations with equal variances against ordered alternatives is obtained and compared with the power functions of usual tests. The Asymptotic efficiency of the Likelihood Ratio test relative to the standard regression test for near alternatives is derived.

The results seem to indicate that in the case of three samples the regression test may have to be preferred. The position is likely to be reversed for more than three samples.

The related results in the corresponding non-parametric problem are also indicated.

3. *Fitting of Orthogonal Polynomials.* *M. V. Pavate and M. Subrahmanyam, Central Tobacco Research Institute, Rajahmundry, South India.*

The use of orthogonal polynomials to experimenters in general and to biologists in particular is well known. In this paper, orthogonal

polynomials have been obtained, using Rao's transformation to obtain a set of uncorrelated variables from mutually correlated variables. The general results are derived for the following three cases, *viz.*,

(1) Equally spaced
(2) Unequally spaced } X-variable.

(3) X-variable with different weights attached.

4. *A Study on Stratification for Sampling of Households in Rural Survey. C. V. Rao, Reserve Bank, Bombay.*

While the choice of sampling unit has a great bearing on the efficiency of estimates, it is known that suitable stratification also results in considerable improvement in the efficiency. Efforts to improve efficiency of estimates through stratification are however generally confined to a stratification of the primary units. The experience of some of the recent surveys shows that significant improvements in the efficiency will also result by stratification of households within the selected villages. For the data on cultivated holdings collected during the First Follow-up Rural Credit Survey conducted by the Reserve Bank the different components of error variance namely the between village contribution and within village contribution have been worked out for the three cases where households are selected (i) without stratification, (ii) after stratifying the households into four groups, the first 10 per cent. of the cultivators according to the descending order of cultivated holdings forming the first group, the next 20 per cent. forming the second group, the next 40 per cent. forming the third group and the last 30 per cent. forming the fourth group and (iii) after stratifying the cultivators in two groups the first 10 per cent. according to the descending order of the cultivated holdings forming the first group and the rest forming the second group.

A comparison of the various components has shown that though the major contribution to the error variance is due to the between village variation, the within village variation also makes a significant contribution to the error variance and in some regions this contribution is almost as important as the between village contribution. It is also seen that while considerable gains in efficiency will result by sampling the households after dividing them into two decile groups, the first 10 per cent. of the households according to descending order of the cultivated holdings forming the first group and the rest forming the second group, further subdivision of the latter group does not lead to any significant increases in the efficiency and therefore not desirable.

5. *On the Relation between the 'Growth Constants' and the Yield of Sugarcane. R. P. Sarker, V. K. Raghavendra and P. S. Nayar, Agri. Met. Division, Meteorological Office, Poona 5.*

Data for fourteen years of sugarcane for the variety POJ. 2878, grown in Poona under Co-ordinated Crop-Weather Scheme, have been examined with a view to investigate the relation between the 'growth constants' and the yield. The final object of this investigation is to forecast yield from the 'growth constants'.

The skew logistic growth curve given by

$$y = d + \frac{K}{1 + e_i a_0 + a_1 x + a_2 x^2 + a_3 x^3}$$

where x is the time co-ordinate and y, the progressive height values was fitted to the data. The 'growth constants', *viz.*, $K + d$, a_0, a_1, a_2, a_3, x, were correlated with yield of canes and a multiple regression equation was worked out for estimating yield from 'growth constants'. The actual yield and the yield estimated by using the regression equation agree well.

6. *Product Decision Procedure for Correlated Characters. M. C. Verma, I.A.R.S., New Delhi.*

Let x, y be two correlated variables and we want to test that the means θ_1 and θ_2 of x, y respectively are:

$$\theta_1 \leqslant 0; \quad \theta_2 \leqslant 0$$

with an additive loss function for this compound decision problem, as used by Lehmann and others. We first confine our attention to Bivariate normal population with known covariance matrix. The risk function for the component problems is simply the II kind of error on one side and the I kind of error is necessarily bounded by a quantity 'α' on the other side ($\theta_1 \leqslant 0$ or $\theta_2 \leqslant 0$).

Under these conditions the product procedure comes out simply as the product of the two standard procedures, *viz.*, ($\bar{x} > k_1$ or $\bar{y} > k_2$). The result can be generalized for more than two characters and investigation is proceeding to include the case of unknown covariance matrix.

7. *On the Definition of Statistical Science and the Divisions Thereof. K. S. Avadhany, Central Statistical Organisation, New Delhi.*

The paper points out three defects in the definitions so far attempted of Statistical Science. (1) They do not bring out in a direct manner

and with the proper emphasis the most important feature of Statistical Science, *viz.*, its applicability in and contact with all other sciences. (2) They do not bring out that there are only two branches of scientific method of study used in all the Sciences, (*a*) the method of the Science Laboratory, (*b*) the Statistical Method, and that the latter method is the one more often met with. (3) They do not do justice to the exalted roles which Statisticians are found filling in recent times. The paper after elaborating on the differences between the two branches of Scientific method, attempts its own definition.

The two branches of Statistical Science, *viz.*, Pure and Applied, are each divided into two main divisions only, and in further subdivisions of these main divisions, it is shown, with the help of a chart, that all the important present-day divisions of statistics can be accommodated.

8. *The Estimation of Total Fruits on Mango Trees by Randomized Branch Sampling Method. K. C. Agarwal, Government Horticultural Research Institute, Saharanpur.*

An experiment has been conducted to evolve a suitable technique for estimating the total number of fruits on a mango tree by counting the fruits borne by a portion of the tree. The twig-wise data of 3 mango trees were taken. The records consisted of the measurement of the circumference of the branch at each stage and the determination of fruits on it. Three sampling schemes, *viz.*, equal probability, probability equal to the number of sub-branches in which a particular branch is subdivided and probability proportional to the square of the circumference of the branch have been tried for 3-stage sampling. The estimates of the 3rd stage sampling with probability proportional to the square of the circumference of the branch have been found to be consistently most efficient.

9. *Extension of Double Sampling Technique to Multi-stage Designs with Varying Probabilities of Selection. J. N. Goswami, I.A.R.S., New Delhi.*

In sample surveys, the unknown mean of a finite population can often be estimated more efficiently by using an auxiliary variable X correlated with Y and whose mean $\bar{X}$ is known. One such estimate is the ratio estimate. When the population mean $\bar{X}$ is not known Sukhatme and Koshal (1959) have used the technique of double sampling. They have derived the results when the double sampling technique is extended to multi-stage designs and where the units are selected with equal probabilities and without replacement at each stage.

The present paper extends the results to the sampling scheme when the units are selected with varying probabilities and with replacement at each stage.

The results have further been extended to stratified sampling and separate and combined ratio estimates have been obtained. It has been shown that combined ratio estimate may provide an estimate which has a negligible bias and whose precision is almost as high as that of the estimate based on separate strata unless the population ratios in the different strata vary considerably.

10. *Optimum Choice of Sampling Units for Multi-character Sample Survey. A. H. Manwani, I.A.R.S., New Delhi.*

The problem of choice of the number of sampling units for more than one character in a sample survey has been considered from the point of view of minimizing a cost function such that the total or the weighted average of the per cent. standard errors or squared per cent. standard errors of the mean of each character is less than a certain fixed quantity. It has ben shown that if weighted function of the per cent. standard errors of 'k' characters, say $K = f(n_1, n_2 \ldots n_k)$ is a convex function of $n_1, n_2 \ldots n_k$ this is true in case of almost all the sampling plans, and if the cost function $C = C(n_1, n_2 \ldots n_k)$ is a concave function, then the optimum sample sizes $n_1^*, n_2^* \ldots n_k^*$ which minimize the cost 'C' w.r.t. the inequality $f(n_1, n_2 \ldots n_k) \leqslant K_0$, are given by the point where the family of surfaces $C = C(n_1 \cdots n_k)$ touches the surface $f(n_1, n_2 \ldots n_k) = K_0$, K_0 being a given fixed quantity.

11. *Effect of Testing Error on Variables Sampling Inspection Plans. B. N. Singh, Indian Standards Institution, New Delhi.*

Sampling inspection by variables assumes that the measurement of the quality characteristic is made by a method which is quite precise. In many cases, however, the test method may not be so precise and as a result, an additional source of error due to the test method is introduced in the various determinations. This paper considers the effect of such testing errors on the known and unknown standard deviation plans and proposes modifications of the same for drawing valid inference.

12. *Diffusion Equations of Modified Branching Process. U. G. Nadkarni, I.A.R.S., New Delhi.*

A Markovian sequence of random variables $z_0, z_1, z_2, \ldots z_n$ representing the number of units in the successive generation is a

modified branching process when the probabilities q_i of fission are dependent upon the size of the set in that generation. For large population at the later stages of this process the approximation for continuous t and x leads to diffusion equations. The characteristic function $\phi(z, t)$, for the frequency function $u(t, x)$, where x represents the size, of the population is obtained and the properties of the process discussed.

PART II

1. *A Note on Fisher's Inequality for Balanced Incomplete Block Designs. V. N. Murty, Central Statistical Organisation, New Delhi.*

All the balanced incomplete block designs with parameters (v, b, r, k, λ) have been classified into four categories according as the Highest Common Factor between v and k is (i) unity (ii) equal to k, (iii) is an integer 'g' between 1 and k such that $v = k \times g$ and (iv) is an integer 'g' between 1 and k. It is shown that for the first three categories, the necessary conditions for the existence of B.I.B. design, *viz.*,

$$bk = vr; \quad r(k - 1) = \lambda(v - 1),$$

imply the inequality $b \geqslant v$ and this inequality is not implied by the above two relations for category (iv).

2. *B.I.B. Designs with one Treatment Missing. K. Seshagiri Rao, College of Science, Nagpur.*

This paper concerns mainly with the problem of analyzing B.I.B. designs when the yields pertaining to a treatment are totally missing, in some particular cases of B.I.B. designs. In these cases it has been shown that the analysis can be carried in ways similar to that of B.I.B. or G.D. or P.B.I.B.D., though with the basic difference that in this case the design is one with non-uniform block sizes. Various B.I.B. designs giving rise to B.I.B. type, G.D. type, and P.B.I.B. type when the yields pertaining to a treatment are deleted have been enumerated. The actual solutions for the Normal equations have been purposefully omitted since the necessary coefficients in these equations have been given and the further procedure is exactly similar to that in the B.I.B. or G.D. or P.B.I.B. as the case may be.

It has been known that by deleting a treatment and all blocks containing it from some B.I.B. designs another B.I.B.D. or a G.D. design can be obtained. By taking all blocks containing a particular treatment and deleting this treatment from these, a second design

which can be called a compliment to the first can be obtained. This technique has been employed in this paper for some B.I.B.D. not discussed so far with the result of obtaining some P.B.I.B. designs in two associate classes besides the above types. All S.B.I.B. designs with $\lambda = 2$ had been exhausted by the above technique. The following two S.B.I.B.D. with $\lambda = 3$ are also studied in detail:

(*a*) $v = b = 11,\ r = k = 6,\ \lambda = 3;$

(*b*) $v = b = 15,\ r = k = 7,\ \lambda = 3.$

By the application of the above technique it has been shown that (*a*) always gives rise to P.B.I.B.D. in two associate classes. The second design also gives rise to a P.B.I.B.D. in two associate classes for certain (common) types of solutions but it has been proved that solutions other than this for this B.I.B.D. cannot exist.

3. *Analysis of* p × q *Factorial Experiment with Unequal Number of Replications by the Use of Multiple Covariance. M. V. Pavate and M. Subrahmanyam, Central Tobacco Research Institute, Rajahmundry, India.*

Yates has suggested a method of weighted analysis for $p \times q$ factorial experiment with disproportionate sub-class numbers. In this paper, the idea of obtaining missing plot values by covariance technique has been extended to obtain the adjusted sums of squares for *A*, *B* and *AB* components, which yields the same results given by Yates for the weighted analysis.

4. *Analysis of Some Experimental Designs when Groups of Treatments have Different Variances. V. Behari, I.A.R.S., New Delhi.*

Ghosh (1960) developed a theory of linear estimation in the case of unequal variances and showed how minimum variance estimates can be obtained which are unbiased and lead to confidence intervals of shortest length.

For randomised block designs Graybill (1954) has given example that the variances for different treatments may be different and Cochran (1947) has suggested that in certain type of experiments it is likely to be so. In this paper the above method has been applied to situations in which groups of treatments have different variances. Estimates of parametric contrasts and their confidence intervals are obtained.

In the case of incomplete block designs, in which block effects are considered as random variables, it has been shown how to transform the observed variables into new variables which can be separated

in two groups. For B.I.B. designs (with parameters: v, b, r, k, λ) estimates and confidence intervals of the contrasts $(t_r - t_s)$ $(r \neq s) = 1, 2, \ldots v)$, *i.e.*, difference of any two treatment effects, have been obtained. This method would be more efficient than the usual intra-block method if the value of σ_t^2/σ_e^2 satisfies a certain inequality relation deduced by the author.

5. *Incomplete Block Designs for Slope-Ratio Assays. G. A. Kulkarni, Institute of Agricultural Research Statistics, New Delhi* 12.

In a paper read before the last meeting of the Society Das and Kulkarni (1960) presented incomplete block designs suitable for parallel line bio-assays. In these designs two contracts due to 'Materials' and Regression (combined slope) which alone enter into the estimation of potency of the test preparations are kept free from block effects. As such the estimation of potency can be obtained as in randomised block designs.

The method of estimation of relative potency from slope-ratio assays is different from that of parallel line assays. Here the two contracts which enter into estimation of relative potency are the two partial regression coefficients of response on the doses from the two preparations. A series of designs suitable for such assays has been obtained. Through these designs the one of the two contrasts can be kept free from block differences.

6. *On Designs Suitable for Varietal-*cum-*Factorial Trials. R. B. D. Sharma, I.A.R.S., New Delhi.*

It is from a suggestion by Dr. Panse that there is the need of designs of experiments for varietal trials at different levels of manurial doses, the present investigation was undertaken. Usually a varietal trial is conducted at some uniform manurial dressing. It is sometimes found that a variety selected from a trial at a particular manurial dressing lodges under higher manurial dose. This points to the necessity of associating different levels of manurial dressings with varietal trials. When the number of manurial treatments is small incomplete block designs augmented by associating each variety with a number of treatments equal to the number of manurial treatments can be adopted. Such designs already exist in literature though the application to meet the present contingency has not been attempted.

Essentially these designs are of the form $(v \times s^n)$, where v denotes the number of varieties and s^n the factorial treatment combinations. Usually v will be very large as compared to s. The usual asymmetrical designs available will thus have very large block size, particularly

because v is large, and as such they are not suitable for these experiments.

In the present investigation we have got several series of such designs involving small block sizes, the number of replications required for these designs is reasonably small.

7. *Construction of Rotatable Designs through Balanced Incomplete Block Designs. V. L. Narasimham, I.A.R.S., New Delhi.*

The Rotatable multifactorial designs were first introduced by Box and Hunter (1957). They constructed such designs with the help of regular geometrical figures. Construction of such designs through geometrical figures proved very difficult when there are four or more number of factors. Subsequently, Das introduced a method of construction of these designs with the help of factorial designs. He could get such designs with reasonably small number of points up to nine factors.

In the present investigation we have taken help of balanced incomplete block designs for the construction of such designs. Through this method second-order rotatable designs can be constructed for any number of factors for which a balanced incomplete block design exists, with reasonably small number of observations. With the help of doubly balanced incomplete block designs or balanced incomplete block design together with its complementary design, third order Rotatable designs, both sequential and non-sequential, could be obtained for up to fifteen factors, though such designs can be obtained for any number of factors through the method.

8. *On Problem Connected with the Asymmetrical Factorial Design. M. N. Das, I.A.R.S., New Delhi.*

Previously there was no systematic method of construction of asymmetrical factorial design. Recently Kishen and Srivastava (1959) and Das (1960) have evolved two methods for constructing these designs. The main problems with these designs are that unless the designs are balanced as also the different affected interactions are estimable mutually independently, they cannot be usefully adopted. Some of the designs available in literature do not satisfy these conditions. Li's (1944) 5×2^2 design in 5 replications is not balanced though the two affected interactions are mutually independently estimable through the design. Again, through the $5 \times 3 \times 2$ design presented as an example by Kishen and Srivastava (1959), the two

affected interactions cannot be estimated independently. As such no error is available for the design.

A further problem is that too many replications are required for balance of such designs. As such we have attempted to obtain some partially balanced designs through which the affected interactions can be estimated mutually independently, requiring at the same time a smaller number of replications.

Again for experiments like $v \times 2^n$ when v is large, too large block sizes are necessary. We have also got some design of this type with reasonably small number of plots per block. These designs are particularly suitable for varietal *cum* manurial trials.

We have also evolved a method of constructing non-resolvable asymmetrical factorial designs of the type 5×2^2 obtained by Shah (1958) through trial and error.

9. *Review of Uniformity Trials.* *K. C. Agarwal, Government Horticultural Research Institute, Saharanpur.*

The up-to-date work on size and shape of plots; number, size and shape of replications; efficiency of different designs and competition effect between the adjacent plots has been fully reviewed. The whole subject has been presented under following heads:

A. *Size and shape of plots for* (i) Cereal crops; (ii) Green and glasshouse experiments; (iii) Vegetable and small fruit plants and (iv) Tree crops.

B. *Size, shape and number of blocks* and

C. *Plot size* verses *number of replications.*

About 100 original papers have been discussed and the history of the development of field experiments in relation to uniformity trials has been completely traced out.

10. *Analysis of Crop Rotation Experiments with Unequal Periods.* *K. N. Agarwal, I.A.R.S., New Delhi.*

Statistical analysis of the data obtained with two crop rotational experiments, one in Jalgaon and other in Dharwar was carried out. In both the experiments there were seven different rotations with different periods. Appropriate constants were fitted to separate out year effects from the plot effects. Estimates of the component of

variances pertaining to permanent differences between plots and those relating to annual differences were obtained. The appropriate standard errors were calculated using these estimates of components of variances for comparison of crop rotations. Analysis of linear regressions of plot yields on years was made to study fertility changes brought out by different rotations. The effects of farm-yard manure on cotton and jowar in the rotations were also examined. Net profits and their standard errors from the different combination of rotations and manuring were calculated.